

# 1 The case of a shorter and more straight forward proof that the fractions: $\left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right|$ are decreasing

## 1.1 How to calculate the $|q_{(2 \cdot t)}|$ from the former q-values

$$q_{(0)} = 1 \text{ and } q_{(2)} = \frac{1}{12}$$

I now calculate  $|q_{(4)}|$  in this way:

$$(2 \cdot 2 + 1) \cdot |q_{(2 \cdot 2)}| = q_{(2)} \cdot q_{(2)}$$

or

$$5 \cdot |q_{(4)}| = q_{(2)} \cdot q_{(2)} = \frac{1}{12} \cdot \frac{1}{12}$$

or

$$|q_{(4)}| = \frac{1}{720}$$

And  $t = 3$  gives  $|q_{(6)}|$  which is calculated in this way:

$$(2 \cdot 3 + 1) \cdot |q_{(2 \cdot 3)}| = q_{(2)} \cdot |q_{(4)}| + |q_{(4)}| \cdot q_{(2)}$$

or

$$7 \cdot |q_{(6)}| = q_{(2)} \cdot |q_{(4)}| + |q_{(4)}| \cdot q_{(2)} = \frac{1}{12} \cdot \frac{1}{720} + \frac{1}{720} \cdot \frac{1}{12}$$

or

$$|q_{(6)}| = \frac{1}{30240}$$

And  $t = 4$  gives  $|q_{(8)}|$  which is calculated in this way:

$$(2 \cdot 4 + 1) \cdot |q_{(2 \cdot 4)}| = q_{(2)} \cdot |q_{(6)}| + |q_{(4)}| \cdot |q_{(4)}| + |q_{(6)}| \cdot q_{(2)}$$

or

$$9 \cdot |q_{(8)}| = q_{(2)} \cdot |q_{(4)}| + |q_{(4)}| \cdot |q_{(4)}| + |q_{(4)}| \cdot q_{(2)} = \frac{1}{12} \cdot \frac{1}{30240} + \frac{1}{720} \cdot \frac{1}{720} + \frac{1}{30240} \cdot \frac{1}{12}$$

or

$$|q_{(8)}| = \frac{1}{1209600}$$

And  $t = 5$  gives  $|q_{(10)}|$  which is calculated in this way:

$$(2 \cdot 5 + 1) \cdot |q_{(2 \cdot 5)}| = q_{(2)} \cdot |q_{(8)}| + |q_{(4)}| \cdot |q_{(6)}| + |q_{(6)}| \cdot |q_{(4)}| + |q_{(8)}| \cdot q_{(2)}$$

or

$$11 \cdot |q_{(10)}| = q_{(2)} \cdot |q_{(8)}| + |q_{(4)}| \cdot |q_{(6)}| + |q_{(6)}| \cdot |q_{(4)}| + |q_{(8)}| \cdot q_{(2)} \\ = \frac{1}{12} \cdot \frac{1}{1209600} + \frac{1}{720} \cdot \frac{1}{30240} + \frac{1}{720} \cdot \frac{1}{30240} + \frac{1}{1209600} \cdot \frac{1}{12}$$

or

$$|q_{(8)}| = \frac{1}{47900160}$$

The q-values seem to get very small very fast.

Maple suggests that the series  $|q_{(2 \cdot t)}$  decreases much like the funktion:

$$\left(\frac{1}{39.5}\right)^t \cdot 2$$

This calculating from former q-values is clearly a job for a program on a computer (as Maple). Even writing the formula gets tiresome. We could instead write it in this way:

$$(2 \cdot 5 + 1) \cdot |q_{(2 \cdot 5)}| = \sum_{s=1}^{5-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

which generalized looks so:

$$(2 \cdot t + 1) \cdot |q_{(2 \cdot t)}| = \sum_{s=1}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

In the following we will need to add  $q_{(2 \cdot t)}$  on both sides of the equation above. That will make it look so:

$$(2 \cdot t + 2) \cdot |q_{(2 \cdot t)}| = q_{(2 \cdot t)} + \sum_{s=1}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

And remembering that  $q_{(0)} = 1$ :

$$(2 \cdot t + 2) \cdot |q_{(2 \cdot t)}| = q_{(0)} \cdot q_{(2 \cdot t)} + \sum_{s=1}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

If now allowing  $s$  in the sum-formula to start being zero we can place  $q_{(0)} \cdot q_{(2 \cdot t)}$  in it and get:

$$(2 \cdot t + 2) \cdot |q_{(2 \cdot t)}| = \sum_{s=0}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

We shall need one equation more in the next section:

This must be the formula to calculate the next q-value after  $|q_{(2 \cdot t)}$

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t+2)}| = \sum_{s=1}^t |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}|$$

But it could be written in this way, which shall be helpful in the following section:

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t+2)}| = \sum_{s=0}^{t-1} |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

To accept this it may help to look at the first and the last addend in the sum. If  $s$  is zero then  $|q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| = |q_{(2)}| \cdot |q_{(2 \cdot t)}$  And If  $s = t - 1$  then

$$|q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| = |q_{(2 \cdot (t-1)+2)}| \cdot |q_{(2 \cdot t-2 \cdot (t-1))}| = |q_{(2 \cdot t)}| \cdot |q_{(2)}|$$

As they should be. Let us recapitulate: For the next section we shall need:

$$(2 \cdot t + 2) \cdot |q_{(2 \cdot t)}| = \sum_{s=0}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

and:

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t+2)}| = \sum_{s=0}^{t-1} |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

## 1.2 The equation of differences

In this section we shall demonstrate that  $|q_{(2 \cdot t)}$  can be written as a sum of the differences between all the former fractions  $\left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right|$  and the endfraction  $\left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right|$  multiplied by the corresponding product  $|q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$ . And which is the same thing, that the following equation is correct:

$$|q_{(2 \cdot t)}| + \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| = 0$$

This equation must be correct:

$$|q_{(2 \cdot t)}| = -(2 \cdot t + 2) \cdot |q_{(2 \cdot t)}| + (2 \cdot t + 3) \cdot |q_{(2 \cdot t)}|$$

and this

$$|q_{(2 \cdot t)}| = -(2 \cdot t + 2) \cdot |q_{(2 \cdot t)}| + \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \cdot (2 \cdot t + 3) \cdot |q_{(2 \cdot t+2)}|$$

Now we have just learnt in the first section how to write  $(2 \cdot t + 2) \cdot |q_{(2 \cdot t)}|$  and  $(2 \cdot t + 3) \cdot |q_{(2 \cdot t+2)}|$  as sums and we will do exactly that:

$$|q_{(2 \cdot t)}| = - \sum_{s=0}^{t-1} |q_{(2 \cdot s)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| + \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \cdot \sum_{s=0}^{t-1} |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

I hope you can accept this small modifications:

$$|q_{(2 \cdot t)}| = - \sum_{s=0}^{t-1} \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| + \sum_{s=0}^{t-1} \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

Now the sums both start with  $s = 0$  and ends with  $s = t - 1$  and the corresponding addends have the product  $|q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$  in common. It must be legal to put them under a common  $\sum$ -sign

$$|q_{(2 \cdot t)}| = - \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

and if we move the sum to the left side, we shall get the wanted equation of differences:

$$|q_{(2 \cdot t)}| + \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| = 0$$

which was to be demonstrated

### 1.3 $\left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right)$ is a decreasing series

In this section we will demonstrate, that  $\left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right)$  starting with  $t = 1$  is decreasing, that is

$$\left( \left| \frac{q_{(2)}}{q_{(4)}} \right| \right) > \left( \left| \frac{q_{(4)}}{q_{(6)}} \right| \right) > \left( \left| \frac{q_{(6)}}{q_{(8)}} \right| \right) > \left( \left| \frac{q_{(8)}}{q_{(10)}} \right| \right) > \dots > \left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) > \dots$$

In the section above we constructed the equation of differences:

$$|q_{(2 \cdot t)}| + \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| = 0$$

The next equation of differences must be this:

$$|q_{(2 \cdot t+2)}| + \sum_{s=0}^t \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t+2)}}{q_{(2 \cdot t+4)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}| = 0$$

Let us formulate a strategy for the proof so that we do not lose the thread in the following. We will try to make these 2 equations look so much alike as possible. The first obstacle is the fraction after the negative sign in the differences. We have in the first equation:

$$\left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right|$$

in the second:

$$\left| \frac{q_{(2 \cdot t + 2)}}{q_{(2 \cdot t + 4)}} \right|$$

We will start removing this:

Let us say, we have found (maybe with the help of Maple) all the fractions  $\left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s + 2)}} \right| \right)$  starting with  $\left( \left| \frac{q_{(2)}}{q_{(4)}} \right| \right)$  and ending with  $\left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| \right)$  to decrease. The last inspection showed that

$$\left( \left| \frac{q_{(2 \cdot t - 2)}}{q_{(2 \cdot t)}} \right| \right) - \left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| \right) > 0$$

but we know nothing about the next difference

$$\Phi = \left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| \right) - \left( \left| \frac{q_{(2 \cdot t + 2)}}{q_{(2 \cdot t + 4)}} \right| \right)$$

I have called it  $\Phi$ .

The first equation of differences above do not tell us anything about  $\Phi$

But the second do:

$$|q_{(2 \cdot t + 2)}| + \sum_{s=0}^t \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s + 2)}} \right| - \left| \frac{q_{(2 \cdot t + 2)}}{q_{(2 \cdot t + 4)}} \right| \right) \cdot |q_{(2 \cdot s + 2)}| \cdot |q_{(2 \cdot t + 2 - 2 \cdot s)}| = 0$$

Without knowing whether  $\Phi$  is positive, negative or zero, it will not make any difference if we write so:

$$|q_{(2 \cdot t + 2)}| + \sum_{s=0}^t \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s + 2)}} \right| - \left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| - \Phi \right) \right) \cdot |q_{(2 \cdot s + 2)}| \cdot |q_{(2 \cdot t + 2 - 2 \cdot s)}| = 0$$

We have just added and subtracted  $\left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| \right)$  in every difference in the sum.

But  $\Phi$  is the same in every of these differences, and hence it can be dragged out and placed in a sum of its own. Though we must not forget to multiply it with the small products. We get:

$$|q_{(2 \cdot t+2)}| + \sum_{s=0}^t \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}| \\ + \sum_{s=0}^t \Phi \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}| = 0$$

or

$$|q_{(2 \cdot t+2)}| + \sum_{s=0}^t \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}| \\ + \Phi \cdot \sum_{s=0}^t |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}| = 0$$

In the first section we constructed the formula for calculating  $|q_{(2 \cdot t+2)}|$

$$(2 \cdot t + 3) \cdot |q_{(2 \cdot t+2)}| = \sum_{s=0}^{t-1} |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

And the next formula for calculating the next q-value ( $|q_{(2 \cdot t+4)}|$ ) in this way must be:

$$(2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = \sum_{s=0}^t |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}|$$

That means we can write our  $\Phi$ -sum from above:

$$\Phi \cdot \sum_{s=0}^t |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}|$$

in this way:

$$\Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}|$$

and our new equation of differences will now look so:

$$|q_{(2 \cdot t+2)}| + \sum_{s=0}^t \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}| \\ + \Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = 0$$

and by the way, after the  $\Phi$  and its small products were dragged out and placed under their own  $\sum$ -sign we can alter the  $t$  in the sum-expression to  $t - 1$  since the the last expression in the sum will be zero from now on:

$$|q_{(2 \cdot t+2)}| + \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t+2-2 \cdot s)}|$$

$$+\Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t + 4)}| = 0$$

Now we have removed the first obstacle.

The second obstacle is:

The small products. The new ones:

$$|q_{(2 \cdot s + 2)}| \cdot |q_{(2 \cdot t + 2 - 2 \cdot s)}|$$

is much smaller than the old ones:

$$|q_{(2 \cdot s + 2)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

A remedy which at first sight might seem rather cosmetic could be to write the new ones in this way:

$$\left| \frac{q_{(2 \cdot t + 2 - 2 \cdot s)}}{q_{(2 \cdot t - 2 \cdot s)}} \right| \cdot |q_{(2 \cdot s + 2)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}|$$

We divide and multiply every product with  $q_{(2 \cdot t - 2 \cdot s)}$

Now the second equation of differences look this way:

$$|q_{(2 \cdot t + 2)}| + \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s + 2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| \right) \cdot \left| \frac{q_{(2 \cdot t + 2 - 2 \cdot s)}}{q_{(2 \cdot t - 2 \cdot s)}} \right| \cdot |q_{(2 \cdot s + 2)}| \cdot |q_{(2 \cdot t - 2 \cdot s)}| \\ + \Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t + 4)}| = 0$$

We could exchange the fraction which varies with  $s$

$$\left| \frac{q_{(2 \cdot t + 2 - 2 \cdot s)}}{q_{(2 \cdot t - 2 \cdot s)}} \right|$$

with this fraction:

$$\left| \frac{q_{(2 \cdot t + 2)}}{q_{(2 \cdot t)}} \right|$$

which do not varies with  $s$

But that will certainly not be a cosmetic alteration: We started this section by asserting that we have found the following true (by calculating it with the help of Maple or in the hand)

$$\left( \left| \frac{q_{(2)}}{q_{(4)}} \right| \right) > \left( \left| \frac{q_{(4)}}{q_{(6)}} \right| \right) > \left( \left| \frac{q_{(6)}}{q_{(8)}} \right| \right) > \left( \left| \frac{q_{(8)}}{q_{(10)}} \right| \right) > \dots > \left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| \right)$$

Then this must be true:

$$\left( \left| \frac{q_{(4)}}{q_{(2)}} \right| \right) < \left( \left| \frac{q_{(6)}}{q_{(4)}} \right| \right) < \left( \left| \frac{q_{(8)}}{q_{(6)}} \right| \right) < \left( \left| \frac{q_{(10)}}{q_{(8)}} \right| \right) < \dots < \left( \left| \frac{q_{(2 \cdot t + 2)}}{q_{(2 \cdot t)}} \right| \right)$$

Which tells us that

$$\left( \left| \frac{q_{(2 \cdot t+2)}}{q_{(2 \cdot t)}} \right| \right)$$

is bigger than all the other fractions

$$\left| \frac{q_{(2 \cdot t+2-2 \cdot s)}}{q_{(2 \cdot t-2 \cdot s)}} \right|$$

it shall replace, since  $a > b$  means  $\frac{1}{a} < \frac{1}{b}$ .

That will cause our sum to get bigger and the zero on the right side can not be kept, will not be true any more. We can remedy this by simply subtracting a positive number equal to the deficiency. We shall call it  $\delta_s$  signifying its dependency of  $s$ .

Now our second equation looks so:

$$|q_{(2 \cdot t+2)}| + \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot \left| \frac{q_{(2 \cdot t+2)}}{q_{(2 \cdot t)}} \right| \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| - \delta_s + \Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = 0$$

or since the positive  $\delta_s$ 's like the  $\Phi$ 's can be dragged out and placed under their own  $\sum$ -sign, and by the way, we shall not need the first  $\delta_s = \delta_0$ , since it must be zero, because here there is no difference between

$$\left| \frac{q_{(2 \cdot t+2-2 \cdot s)}}{q_{(2 \cdot t-2 \cdot s)}} \right|$$

and

$$\left| \frac{q_{(2 \cdot t+2)}}{q_{(2 \cdot t)}} \right|$$

for  $s = 0$ :

$$|q_{(2 \cdot t+2)}| + \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot \left| \frac{q_{(2 \cdot t+2)}}{q_{(2 \cdot t)}} \right| \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}| - \sum_{s=1}^{t-1} \delta_s + \Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = 0$$

Further more we can drag the fraction  $\left| \frac{q_{(2 \cdot t+2)}}{q_{(2 \cdot t)}} \right|$  being no longer dependent of  $s$  out of and in the front of the sum of differences:

$$|q_{(2 \cdot t+2)}| + \left| \frac{q_{(2 \cdot t+2)}}{q_{(2 \cdot t)}} \right| \cdot \sum_{s=0}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

$$-\sum_{s=1}^{t-1} \delta_s + \Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = 0$$

If we now multiply everything in the equation with the fraction  $\left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right|$ , the first expression on the left side of the equation  $|q_{(2 \cdot t+2)}$  multiplied by the fraction will transform to

$$|q_{(2 \cdot t)}|$$

and the fraction before  $\sum$ -sign will disappear, getting 1 when multiplied by its reciprocal partner

and we will still be able to keep our zero on the right side, since

$$\left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \cdot 0 = 0$$

. We get:

$$|q_{(2 \cdot t)}| + \sum_{s=1}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

$$- \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \cdot \sum_{s=1}^{t-1} \delta_s + \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \cdot \Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = 0$$

This was the second obstacle.

Is this not beautiful. Remember the first equation of differences? The first 2 expression here:

$$|q_{(2 \cdot t)}| + \sum_{s=1}^{t-1} \left( \left| \frac{q_{(2 \cdot s)}}{q_{(2 \cdot s+2)}} \right| - \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \right) \cdot |q_{(2 \cdot s+2)}| \cdot |q_{(2 \cdot t-2 \cdot s)}|$$

is equal to the left side of that equation and must be zero. What have we left?

$$- \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \cdot \sum_{s=1}^{t-1} \delta_s + \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right| \cdot \Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = 0$$

We can divide on both side of this new equation with the fraction:  $\left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t+2)}} \right|$  and we get

$$-\sum_{s=0}^{t-1} \delta_s + \Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = 0$$

or

$$\Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t+4)}| = \sum_{s=1}^{t-1} \delta_s$$

We did not know the sign of  $\Phi$  but we do know the sign of our  $\delta_s$ 's. They are all positive numbers hence the sum of them must be positive and so our  $\Phi$ . We can write:

$$\Phi \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t + 4)}| = \sum_{s=1}^{t-1} \delta_s$$

or remembering:

$$\Phi = \left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| - \left| \frac{q_{(2 \cdot t + 2)}}{q_{(2 \cdot t + 4)}} \right| \right)$$

We can write

$$\left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| - \left| \frac{q_{(2 \cdot t + 2)}}{q_{(2 \cdot t + 4)}} \right| \right) \cdot (2 \cdot t + 5) \cdot |q_{(2 \cdot t + 4)}| = \sum_{s=1}^{t-1} \delta_s$$

or

$$\left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| - \left| \frac{q_{(2 \cdot t + 2)}}{q_{(2 \cdot t + 4)}} \right| \right) = \frac{\sum_{s=1}^{t-1} \delta_s}{(2 \cdot t + 5) \cdot |q_{(2 \cdot t + 4)}|}$$

The left side of this equation must certainly be positive because the sum of deltas are and this sum is divided by a positive number and the q-value:  $|q_{(2 \cdot t + 4)}|$ , which is secured positive by the numeric sign.

We can conclude that if we have found a segment of the series  $\left( \left| \frac{q_{(2 \cdot t)}}{q_{(2 \cdot t + 2)}} \right| \right)$  starting with  $\left( \left| \frac{q_{(2)}}{q_{(4)}} \right| \right)$  (this is a necessary condition) decreasing then the next fraction is smaller than the last member of the controlled segment, Which was to be demonstrated.

By the way the first 2 fractions are fulfilling the conditions

$$\left( \left| \frac{q_{(2)}}{q_{(4)}} \right| \right) = 60$$

and

$$\left( \left| \frac{q_{(4)}}{q_{(6)}} \right| \right) = 42$$

According to the demonstration above

$$\left( \left| \frac{q_{(6)}}{q_{(8)}} \right| \right)$$

should be smaller.

It is: being 40

And I do not need Maple to calculate this :)

This is a classic proof of induction.